Activity description

In this activity students fit functions to graphs (linear and quadratic).
They compare models and comment on their suitability

Suitability

Level 2 (Intermediate/Higher)
The activity could also be used early in a Level 3 (Advanced) course.

Time

2–3 hours

Resources and equipment

Student information sheet and worksheet, spreadsheet, answer sheet, graph paper

Optional: computers, graphic calculators, slideshow.

Key mathematical language

Linear, quadratic, gradient, simultaneous equation

Notes on the activity

This activity uses the same data as another activity called 'Road Test' which you may have used earlier.

The focus in this activity is on finding linear and quadratic functions to model the data. It is assumed that students will already have some knowledge of linear and quadratic functions and their graphs.

The slideshow could be used to introduce the activity and remind students of the earlier work if they have done this.

The first question on the student sheets requires students to find a linear function using the intercept and gradient of a linear graph. For later questions they need to be able to recognise the shape of quadratic curves, and know how to solve simultaneous equations.

During the activity

After question 1, the use of computer software or graphic calculators would allow students to concentrate on the maths rather than the mechanics of drawing curved graphs.

The data is provided on a spreadsheet as well as being given in the student sheets.

Points for discussion

Student sheet discussion questions

In the slideshow and student sheets the tables of data are followed by questions. These should encourage students to look carefully at patterns in the data, and help to generate class discussion.

In table 1, the speedometer speed (u) is greater than the true speed (v), but the difference is not always the same. The rough rule connecting the two sets of speeds is that u is roughly 10% greater than v or $u = 1.1v$.
A more accurate rule can be found using a graph.

It is more difficult to know what sort of function will fit the table 2 data because the function is not linear.

Questions to help students reflect on their work

The last slide has questions to help students reflect on the work they have done. These are included on the student sheets and repeated below.

- Which method of finding a model do you like best?
- Which method do you think is the most accurate?
- Would using a cubic or higher order function be better? (Some cubic models are given in the spreadsheet 'Modelling a test drive answers' and could be displayed for comments.)
- What difference would it make if you took into account inaccuracies in the data you have been given? – relevant for scientists and engineers.

As well as the points given above, you may have students learning to drive and who know about the braking distances given in the Highway Code. Are those distances the same?

Extensions

Drawing and interpreting graphs

The same data can be used for practice in drawing and interpreting graphs. See the Nuffield 'Road test' resources.

Finding and interpreting gradients

The gradient of a graph of 'True speed against time' can be found by drawing tangents at various points on it. These can used to estimate the acceleration of the car at these times.

Note that you need to convert the speed into m s^{-1} in order to give the acceleration in m s^{-2}.

Area under a curve

The area under a graph of 'True speed against time' can be used to estimate the distance travelled by the car. This can be done using the whole curve or sections of it.

Note that you need to convert the speed into m s^{-1} in order to give the distance in metres.

Answers

The spreadsheet 'Modelling a test drive answers' contains the relevant data and graphs. If possible project the graphs to aid class discussion.

1 Indicated speed against true speed

a The graph of 'Indicated speed against true speed' is given on the spreadsheet 'Indicated–True speed'.

c The assumption is that the indicated speed is zero when the car is at rest.

d Gradient is approximately 1.1 (2 sf).
This is the extra speed indicated for each extra mile per hour of true speed.

e $u = 1.1v$

f 110 mph (2 sf)

g 10%

h The true speed is less than the speed indicated, so you can travel at indicated speeds slightly greater than the speed limit without actually breaking the speed limit. Manufacturers err on the side of caution in calibrating speedometers, so that drivers cannot claim faulty instruments when charged with speeding.

Extension

The graph and trend line are given in the sheet 'Indicated–True Trend'.

The equation relating v and u suggested by the trend line is $u = 1.1v + 1$

The main difference is the fact that this does not pass through the origin. The intercept value of 1 suggests that the speedometer indicates a speed of 1 mph when the car is at rest.

2 Braking

a, b The graph of 'Distance against true speed' is given in spreadsheet 'Brakes dist-speed'.

c If the speed is zero, the car is already at rest, so the distance taken to come to rest is zero.

d $k = 0.0137$ (3 sf)

e The graph on spreadsheet 'Brakes-Quad Models' includes the curve $d = 0.0137v^2$.

f The graph of the model passes through the origin and the last data point (as it is bound to do), but predicts values of d that are too high for other values of v between these points.

g A full list of k values found by substituting data values is given in this table.

Point used	(30, 9.5)	(50, 27.5)	(70, 52.4)	(85, 98.7)
Value of k (3 sf if not exact)	0.0106	0.011	0.0107	0.0137

The graph on spreadsheet 'Brakes-Quad models' includes the curve $d = 0.0106v^2$ which gives a good fit for values of v up to 70. The remaining curves (with k values found from other data points) are very similar.

Extension

The graph with quadratic trendline is given In sheet 'Brakes-Trend'.

The function suggested by the trendline is $d = 0.017v^2 - 0.36\,v + 1.4$.

There are problems with this model. In particular, it predicts negative distances for coming to rest from speeds near to 10 mph, and a positive distance when the speed is zero.

3 Acceleration from rest

a The graph of 'True speed against time' is given on sheets 'True speed-time' and 'Acceln Graph'

b Initially the true speed is zero.

d Substituting $t = 0$, $v = 0$ into the equation gives $c = 0$.

f $80 = 171.61a + 13.1b$

g $a = -0.232$, $b = 9.15$ (3 sf)

h Spreadsheet 'Acceln-Quad Models' shows the curve $v = 9.15t - 0.232t^2$

i The graph of the model passes through the origin and the last two data points (as it is bound to do), but predicts values of v that are too low for $0 < t < 13.1$.

j Using the first and last data pairs gives the model $v = 14.1t - 0.475t^2$

k The graph of $v = 14.1\text{t} - 0.475t^2$ is included in 'Acceln-Quad Models'.

This model is not as good as the previous model because of its maximum point. It predicts a decrease in speed in the later part of the graph, whereas the speed is actually increasing.

Extension

The Excel graph and trend lines are given on sheet 'Acceln-Trend'.

The equation of the quadratic trend line is $v = -0.273t^2 + 9.59t + 6.07$
The main problems with this model are the maximum point (again predicting decreasing speed in the later section) and the positive intercept which suggests that the car does not start from rest. The cubic trend line, $v = 0.0177t^3 - 0.807t^2 + 13.5t + 1.17$ is a much better fit.